Lecture 10. System Modeling Fundamentals

The purpose of the lecture: introduction to the conceptual foundations of systems modeling.

Lecture plan:
Introduction
1 System Modeling Fundamentals
Conclusion

Keywords: model, modeling, object, physical system, software, stability, word, measure, place, mathematical structures, cognitive, pragmatic, instrumental, cognitive model, knowledge representation, pragmatic model, instrumental model, sets, universal methods, meaning, static, static model, Newton's law, dynamic, parameter, dynamic model, discrete, discrete model, continuous, simulation, cost, simulation model, deterministic, stochastic, stochastic model, functional, set-theoretic, membership relation, set-theoretic model, logical, game , game, matrix game, game theory, algorithmic, computation model, algorithm, series sum, structural, structural model, graph, hierarchical, network, cast operation, graph arc, network model, network graph, arc, linguistic, concatenation operation, operation assignment, visual, full-scale, geometric, layout, geometric model, Circle, plane, cellular automaton, automaton, dynamical system, geometry, evolution, field, computation, cellular automaton model, diffusion, unit, combinations, distance, probability, cellular autonomous model, fractal, connection, dimension, fractal model, segment, self-similarity, property, life cycle, isomorphism.

Contents of the lecture:

Introduction

The basic concepts of systems modeling, system types and properties of models, the life cycle of modeling (modeled system) are considered.

1 System Modeling Fundamentals

Model and modeling are universal concepts, attributes of one of the most powerful methods of cognition in any professional field, cognition of a system, process, phenomenon.

Models and modeling bring together experts from different fields working on solving cross-subject problems, regardless of where the model and the results of the modeling will be applied. The type of the model and the methods of its research depend more on the information and logical connections of the elements and subsystems of the modeled system, resources, connections with the environment used in modeling, and not on the specific nature, specific content of the system.

Models, especially mathematical ones, also have didactic aspects – the development of a model style of thinking that allows one to delve into the structure and internal logic of the modeled system.

Building a model is a systemic task that requires analysis and synthesis of initial data, hypotheses, theories, and knowledge of specialists. The systematic approach allows not only to build a model of a real system, but also to use this model to assess (for example, management efficiency, functioning) of the system.

Model – an object or description of an object, a system for replacing (under certain conditions, sentences, hypotheses) one system (i.e. the original) with another system for better studying the original or reproducing any of its properties. A model is the result of mapping one structure (studied) to another (little-studied). By mapping a physical system (object) to a mathematical system (for example, the mathematical apparatus of equations), we obtain a physical and mathematical model of the system or a mathematical model of a physical system. Any model is built and investigated under certain assumptions and hypotheses.

Example. Consider a physical system: a body of mass m rolling down an inclined plane with an acceleration a, which is acted upon by a force F. Investigating such systems, Newton obtained the mathematical relation: $F = ma$. This is a physical and mathematical model of a system or a mathematical model of a physical system. When describing this system (building this model), the following hypotheses were accepted: 1) the surface is ideal (that is, the coefficient of friction is zero); 2) the body is in a vacuum (i.e. the air resistance is zero); 3) body weight is unchanged; 4) the body moves with the same constant acceleration at any point.

Example. The physiological system – the human circulatory system – obeys certain laws of thermodynamics. Describing this system in the physical (thermodynamic) language of balance laws, we obtain a physical, thermodynamic model of a physiological system. If we write down these laws in mathematical language, for example, write down the corresponding thermodynamic equations, then we will already get a mathematical model of the circulatory system. Let's call it a physiological-physical-mathematical model or a physical-mathematical model.

Example. A set of enterprises operates in the market, exchanging goods, raw materials, services, information. If we describe economic laws, the rules of their interaction in the market with the help of mathematical relations, for example, a system of algebraic equations, where the unknowns will be the values of profit obtained from the interaction of enterprises, and the coefficients of the equation will be the values of the intensities of such interactions, then we will obtain a mathematical model of the economic system, i.e. e. economic and mathematical model of the system of enterprises in the market.

Example. If a bank has developed a lending strategy, was able to describe it with the help of economic and mathematical models and predict its lending tactics, then it has greater stability and vitality.

The word "model" (lat. Modelium) means "measure", "method", "resemblance to some thing."

Modeling is based on the mathematical theory of similarity, according to which absolute similarity can take place only when one object is replaced by another exactly the same. When modeling most systems (with the possible exception of modeling some mathematical structures by others), absolute similarity is impossible, and the main goal of modeling is that the model should reflect well the functioning of the modeled system.

The models, if we ignore the areas, their spheres of application, are of three types: cognitive, pragmatic and instrumental.

A cognitive model is a form of organization and presentation of knowledge, a means of connecting new and old knowledge. The cognitive model, as a rule, is fitted to reality and is a theoretical model.

The pragmatic model is a means of organizing practical actions, a working representation of the goals of the system for its management. The reality in them is adjusted to a certain pragmatic model. These are, as a rule, applied models.

An instrumental model is a means of building, researching and / or using pragmatic and / or cognitive models.

Cognitive ones reflect existing, and pragmatic ones – although not existing, but desirable and, possibly, feasible relationships and connections.

In terms of level, "depth" of modeling, models are:

- ✓ *empirical - based on empirical facts, dependencies;*
- ✓ *theoretical - based on mathematical descriptions;*
- ✓ *mixed, semi-empirical - based on empirical dependencies and mathematical descriptions.*

The modeling problem consists of three tasks:

- ✓ *building a model (this task is less formalized and constructive, in the sense that there is no algorithm for building models);*
- ✓ *model study (this task is more formalized, there are methods for studying various classes of models);*
- ✓ *use of the model (constructive and concrete task).*

Model M, describing the system $S(x_1, x_2, ..., x_n; R)$, has the form: $M = (z_1, z_2, ..., z_m; Q)$, where $z_i \in Z$, $i = 1, 2, ..., n$, Q, R – sets of relations over X – set of input, output signals and system states, Z – set of descriptions, representations of elements and subsets of X.

The scheme for constructing a model M of system S with input signals X and output signals Y is shown in Fig. 10.1.

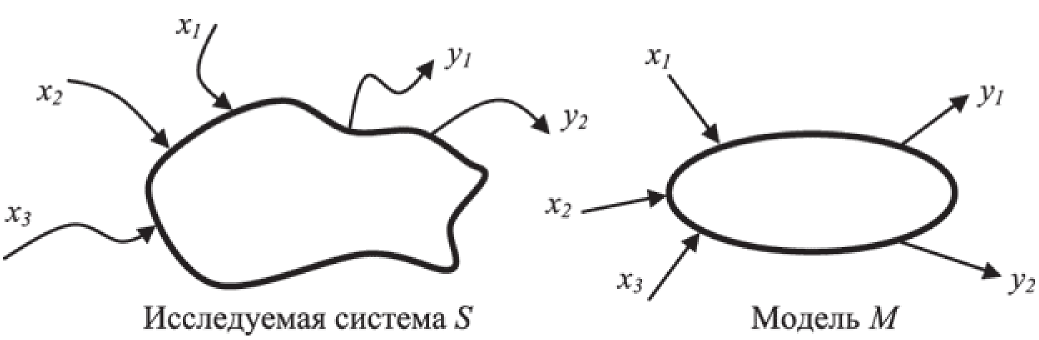

Figure 10.1. Model building scheme

If signals from X arrive at the input M and signals Y appear at the input, then a law is given, a rule f for the functioning of the model, system.

Modeling is a universal method of obtaining, describing and using knowledge. It is used in any professional activity. In modern science and technology, the role and significance of modeling is increasing, actualized by the problems, successes of other sciences. Modeling real and nonlinear systems of animate and inanimate nature allows us to throw bridges between our knowledge and real systems, processes, including mental ones.

Models are classified according to various criteria. We will use the most simple and practical one.

A model is called static if there is no temporal parameter among the parameters involved in its description. The static model at each moment of time gives only a "photograph" of the system, its slice.

Example. Newton's law $F = am$ is a static model of a material point of mass m moving with acceleration a. This model does not account for the change in acceleration from one point to another.

A model is dynamic if there is a time parameter among its parameters, i.e. it displays the system (processes in the system) over time.

Example. Model $S = gt^2/2$ – dy namic model of the path in free fall of the body. Dynamic model like Newton's law: $F(t) = a(t)m(t)$. An even better form of Newton's dynamic model is $F(t) = s''(t)m(t)$.

A model is discrete if it describes the behavior of the system only at discrete times.

Example. If we consider only $t = 0, 1, 2, \ldots, 10$ (sec), then the model $S_t = gt^2/2$ or the numerical sequence $S_0 = 0$, $S_1 = g/2$, $S_2 = 2g$, $S_3 = 9g/2$, …, $S_{10} = 50g$ can serve as a discrete model of the motion of a freely falling body.

A model is continuous if it describes the behavior of the system for all times from a certain time interval.

Example. The model $S = gt^2/2$, $0 < t < 100$ is continuous over the time interval (0; 100).

A simulation model if it is designed to test or study possible paths of development and behavior of an object by varying some or all of the parameters of the model.

Example. Let the model of the economic system for the production of goods of two types 1 and 2, respectively, in the number of x_1 and x_2 units and the cost of each unit of goods a_1 and a_2 at the enterprise is described as a ratio: $a_1x_1 + a_2x_2 = S$, where S is the total value of all products produced by the enterprise (types 1 and 2). It can be used as a simulation model by which it is possible to determine (vary) the total cost S depending on certain values of the volumes of goods produced.

The model is deterministic if each input parameter set corresponds to a well-defined and uniquely definable set of output parameters; otherwise, the model is non-deterministic, stochastic (probabilistic).

Example. The above physical models are deterministic. If in the model $S = gt^2/2$, $0 < t < 100$, we would take into account a random parameter – a gust of wind with a force p when the body falls, for example, like this: $S(p) = g(p)t^2/2$, $0 < t < 100$, then we would get a stochastic model (no longer free!) Fall.

A model is functional if it can be represented as a system of some functional relations.

Example. Continuous, deterministic Newton's law and the model of production of goods (see above) are functional.

A model is set-theoretic if it is representable using some sets and relations of belonging to them and between them.

Example. Let the set X = {Nikolay, Peter, Nikolaev, Petrov, Elena, Ekaterina, Mikhail, Tatiana} and relations be given: Nikolay is Elena's husband, Ekaterina is Peter's wife, Tatyana is the daughter of Nikolai and Elena, Mikhail is the son of Peter and Ekaterina, family Michael and Peter are friends with each other. Then the set X and the set of the listed relations Y can serve as a set-theoretic model of two friendly families.

The model is logical if it can be represented by predicates, logical functions.

Example. The combination of two logical functions of the form: $z = x \wedge y \vee x \wedge y, p = x \wedge y$ can serve as a mathematical model of a one-digit adder.

A game model, if it describes, implements a certain game situation between the participants in the game (persons, coalitions).

Example. Let player 1 be a bona fide tax inspector, and player 2 an unscrupulous taxpayer. There is a process (game) on tax evasion (on the one hand) and on revealing tax evasion (on the other hand). Players choose natural numbers i and j (i, j ≤ n), which can be identified, respectively, with the penalty of player 2 for non-payment of taxes upon detection of the fact of non-payment by player 1 and with the temporary benefit of player 2 from tax evasion (in the medium and long-term the penalty for concealment can be much more severe). Consider a matrix game with payoff matrix of order n. Each element of this matrix A is determined by the rule $a_{ij} = | i - j|$. The game model is described by this matrix and the dodge and catch strategy. This game is antagonistic, non-coalitional (for now, we will understand the concepts formalized in the mathematical theory of games meaningfully, intuitively).

A model is algorithmic if it is described by some algorithm or a set of algorithms that determines its functioning, development. The introduction of such, at first glance, an unusual type of models (indeed, it seems that any model can be represented by an algorithm for its study), in our opinion, is quite justified, since not all models can be investigated or implemented algorithmically.

Example. The model for calculating the sum of an infinite decreasing series of numbers can be an algorithm for calculating the finite sum of a series to a certain specified degree of accuracy. An algorithmic model of the square root of the number x can be an algorithm for calculating its approximate, arbitrarily accurate value according to a well-known recurrent formula.

A model is structural if it is represented by a data structure or data structures and relationships between them.

Example. A structural model can be a description (tabular, graph, functional, or other) of the trophic structure of an ecosystem. Build such a model (one of them was given above).

A graph model if it can be represented by a graph or graphs and relations between them.

A model is hierarchical (tree-like) if it is represented by some hierarchical structure (tree).

Example. To solve the problem of finding a route in a search tree, you can build, for example, a tree model (Fig.10.2):

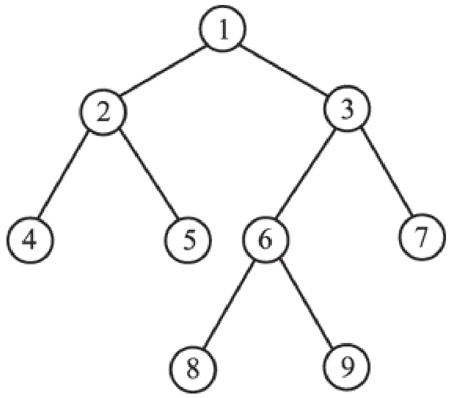

Figure 10.2. Hierarchical structure model

A model is network if it can be represented by some network structure.

Example. The construction of a new home includes the operations shown in the following table.

Table 10.1 House construction work table

No.	***Operation***	***Execution time (days)***	***Previous operations***	***Graph arcs***
1	clearing the area	1	no	
2	laying the foundation	4	clearing the area (1)	1 – 2
3	wall construction	4	laying the foundation (2)	2 – 3
4	electrical installation	3	wall construction (3)	3 – 4
5	plastering work	4	electrical installation (4)	4 – 5
6	improvement of the territory	6	wall construction (3)	3 – 6
7	finishing work	4	plastering work (5)	5 – 7
8	roof deck	5	wall construction (3)	3 – 8

The network model (network diagram) of building a house is shown in Fig. 10.3.

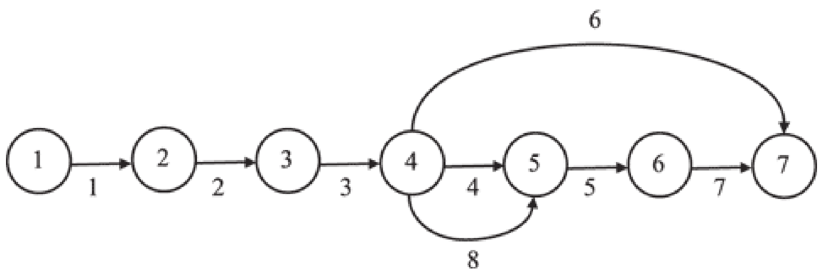

Figure 10.3. Construction work network schedule

Two works corresponding to arc 4-5 are parallel, they can either be replaced with one representing a joint operation (installation of electrical wiring and roof decking) with a new duration of 3 + 5 = 8, or you can enter a fictitious event on one arc, then arc 4-5 will take view.

The model is linguistic, linguistic, if it is represented by some linguistic object, formalized linguistic system or structure. Sometimes such models are called verbal, syntactic, etc.

Example. Traffic rules - linguistic, structural model of traffic and pedestrian traffic on the roads. Let B be the set of generating stems of nouns, C – the set of suffixes, P – adjectives, "+" – the operation of concatenation of words, ": =" – the operation of assignment, "⟹" – the operation of inference (deducibility of new words), Z – the set of meanings (semantic) adjectives. Language model M word formation: <zi> ⟸ <pi>: = <bi> + <si>. With bi - "рыб(а)", si - "н(ый)", we get by this model pi - "рыбный", zi - "приготовленный из рыбы".

The model is visual if it allows you to visualize the relationships and connections of the modeled system, especially in dynamics.

Example. A visual model of an object is often used on a computer screen, for example, a keyboard in a keyboard training program.

A full-scale model if it is a material copy of the modeling object.

Example. Globe is a full-scale geographic model of the globe.

The model is geometric, graphic, if it is represented by geometric images and objects.

Example. The house model is a full-scale geometric model of a house under construction. A polygon inscribed in a circle gives a model of the circle. It is she who is used when depicting a circle on a computer screen. A straight line is a model of a number axis, and a plane is often depicted as a parallelogram.

A model is cellular automaton if it represents a system using a cellular automaton or a system of cellular automata. A cellular automaton is a discrete dynamic system, an analogue of a physical (continuous) field. Cellular automata geometry is an analogue of Euclidean geometry. An indivisible element of Euclidean geometry is a point, on the basis of which segments, lines, planes, etc. are constructed. An indivisible element of the cellular automaton field is a cell, on the basis of which clusters of cells and various configurations of cellular structures are built. This is the "world" of some automaton, executor, structure. The cellular automaton is represented as a uniform network of cells ("cells") of this field. The evolution of a cellular automaton unfolds in a discrete space - a cellular field. Such cellular fields can be material-energy-informational. The laws of evolution are local, i.e. the dynamics of the system is determined by a set unchanging set of laws or rules, according to which a new cell of evolution and its material-energy-informational characteristics are calculated, depending on the state of its surrounding neighbors (the rules of neighborhood, as already mentioned, are set). The change of states in the cellular automaton field occurs simultaneously and in parallel, and time goes discretely. Despite the seeming simplicity of their construction, cellular automata can exhibit varied and complex behavior. Recently, they have been widely used in modeling not only physical, but also socio-economic processes.

Cellular automata (fields) can be one-dimensional, two-dimensional (with cells in the plane), three-dimensional (with cells in space), or multidimensional (with cells in multidimensional spaces).

Example. The classic cellular automaton model is the game "Life" by John Conway. She is described in many books. We will consider another cellular automaton model of environmental pollution, the diffusion of a pollutant in a certain environment. A 2D cellular automaton (on a plane) for modeling environmental pollution can be generated by the following rules:

- *the plane is divided into identical cells: each cell can be in one of two states: state 1 - there is a diffusing pollutant particle in it, and state 0 - if it is absent;*
- *the cell field is divided into 2x2 blocks in two ways, which we will call even and odd partitions (an even partition has an even number of points or cells of the field in a cluster or block, an odd block has an odd number);*
- *at the next step of evolution, each block of the even partition is rotated (according to the specified rule of contamination propagation or the generated distribution of random numbers) by a given angle (the direction of rotation is chosen by the random number generator);*
- *a similar rule is defined for odd-partitioned blocks;*
- *the process continues until some point or until the environment is purified.*

Let the unit of time be the step of the cellular automaton, the unit of length be the size of its cell. If we sort out all possible combinations of rotations of blocks of an even and odd partition, we see that in one step a particle can move along each of the coordinate axes at a distance of 0, 1, or 2 (without taking into account the direction of displacement) with probabilities, respectively, $p_0 = 1/4$, $p_1 = 1/2$, $p_2 = 1/4$. The probability of a particle hitting a given point depends only on its position at the previous moment of time; therefore, we consider the motion of a particle along the x(y) axis as random.

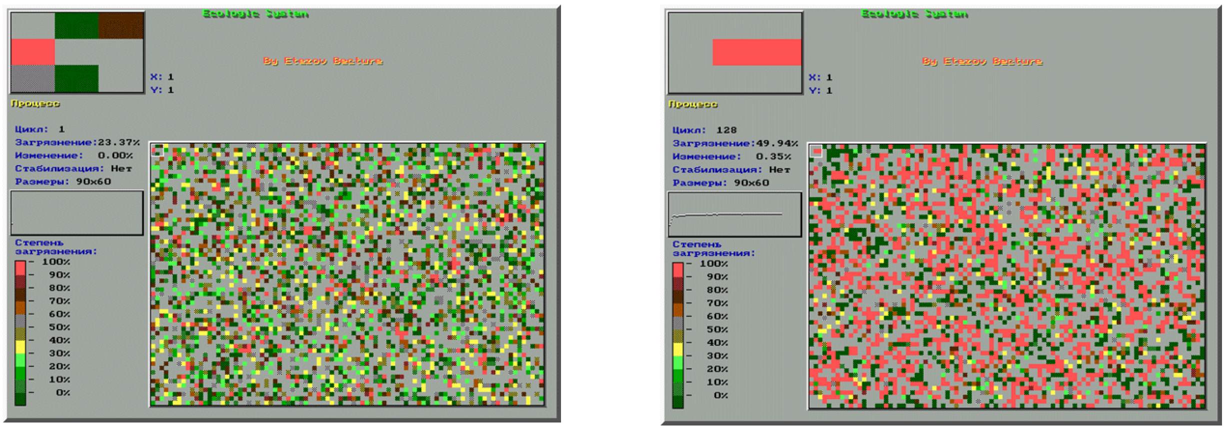

Figure 10.4. Window on the right - the state of the cell field (in the upper - the initial, slightly contaminated, in the lower - after 120 cycles of contamination), in the upper left corner - "Microscope", increasing the cluster of the field, in the middle left - the contamination dynamics graph, in the lower left - contamination indicators

In fig. 10.4 – fragments of the work of the program of the cellular automaton model of contamination of the cellular ecological environment (cell sizes are increased).

The model is fractal if it describes the evolution of the modeled system by the evolution of fractal objects. If a physical object is homogeneous (solid), i.e. there are no cavities in it, we can assume that the density does not depend on the size. For example, as R increases to 2R, the mass will increase R^2 times (circle) and R^3 times (ball), i.e. $M(R) \sim R^n$ (relationship between mass and length), n is the dimension of space. An object whose mass and size are related by this ratio is called "compact". Its density

$$\rho \sim \frac{M}{R^n} \sim R^0 - const$$

If an object (system) satisfies the relation $M(R) \sim R^{f(n)}$, where $f(n) < n$, then such an object is called fractal. Its density will not be the same for all R values, but scaled like this:

$$\rho(R) \sim \frac{M(R)}{R^n} \sim R^{f(n)-n}.$$

Since $f(n) - n < 0$, the density of a fractal object decreases with increasing size, and $\rho(R)$ is a quantitative measure of the thinness, branchiness (structuredness) of the object.

Example. An example of a fractal model is the Cantor set. Consider [0; 1]. Divide it into 3 parts and discard the middle segment. Divide the remaining 2 gaps into three parts again and discard the middle gaps, etc. We get a set called the Cantor set. In the limit, we get an uncountable set of isolated points (Fig.10.5)

Figure 10.5. Cantor set for 3 divisions

It can be shown that if n is the dimension of the Cantor set, then $n = \ln 2/\ln 3 \approx 0.63$, i.e. this object (fractal) does not yet consist only of isolated points, although it does not already consist of a line segment. Fractal objects are self-similar if they look the same at any spatial scale, are scale-invariant, fragments of the structure are repeated at certain spatial intervals. Therefore, they are very well suited for modeling irregularities, since they allow describing (for example, discrete models) the evolution of such systems for any moment in time and on any spatial scale.

Self-similarity is found in a wide variety of objects and phenomena.

Example. Tree branches, snowflakes, economic systems (Kondratyev waves), mountain systems are self-similar.

The fractal model is usually used when a real object cannot be represented in the form of a classical model, when we are dealing with nonlinearity (multivariance of development paths and the need for choice) and indeterminacy, chaos and irreversibility of evolutionary processes.

The type of model depends on the informational essence of the modeled system, on the connections and relationships of its subsystems and elements, and not on its physical nature.

Example. Mathematical descriptions (models) of the dynamics of the epidemic of an infectious disease, radioactive decay, the acquisition of a second foreign language, the release of products of a manufacturing enterprise, etc. are the same in terms of their description, although the processes are different.

The boundaries between models of different types or the assignment of a model to one type or another are often very arbitrary. We can talk about different modes of using the models – imitation, stochastic, etc.

The model includes: object O, subject (optional) A, task Z, resources B, modeling environment C: M = <O, Z, A, B, C>.

The main properties of any model:

- ✓ *purposefulness – the model always reflects a certain system, i.e. has a purpose;*
- ✓ *finiteness – the model reflects the original only in a finite number of its relations and, in addition, the modeling resources are finite;*
- ✓ *Simplicity – the model displays only the essential aspects of the object and, in addition, should be easy to study or reproduce;*
- ✓ *approximation – the reality is shown by the model roughly or approximately;*
- ✓ *adequacy – the model must successfully describe the modeled system;*
- ✓ *visibility, visibility of its main properties and relationships;*
- ✓ *availability and manufacturability for research or reproduction;*
- ✓ *informativeness – the model should contain sufficient information about the system (within the framework of the hypotheses adopted when building the model) and should make it possible to obtain new information;*
- ✓ *saving the information contained in the original (with the accuracy of the hypotheses considered when constructing the model);*
- ✓ *completeness – the model must take into account all the basic connections and relationships necessary to ensure the goal of modeling;*
- ✓ *stability – the model should describe and ensure the stable behavior of the system, even if it is initially unstable;*
- ✓ *integrity – the model implements some system (i.e. the whole);*
- ✓ *isolation – the model takes into account and displays a closed system of necessary basic hypotheses, connections and relationships;*
- ✓ *adaptability – the model can be adapted to various input parameters, environmental influences;*
- ✓ *controllability (imitation) – the model must have at least one parameter, the changes of which can imitate the behavior of the modeled system under various conditions;*
- ✓ *evolving – the possibility of developing models (previous level).*

Life cycle of the simulated system:

+ *collection of information about the object, hypothesis, pre-model analysis;*

+ *design of the structure and composition of models (submodels);*
+ *construction of model specifications, development and debugging of individual sub-models, assembly of the model as a whole, identification (if necessary) of model parameters;*
+ *model research - the choice of a research method and the development of a modeling algorithm (program);*
+ *study of the adequacy, stability, sensitivity of the model;*
+ *assessment of modeling tools (spent resources);*
+ *interpretation, analysis of modeling results and establishment of some cause-and-effect relationships in the system under study;*
+ *generation of reports and design (national economic) solutions;*
+ *refinement, modification of the model, if necessary, and return to the system under study with new knowledge obtained using the model and modeling.*

Modeling is a system analysis method. But often in system analysis with a model approach to research, one methodical error can be made, namely, the construction of correct and adequate models (submodels) of the subsystems of the system and their logically correct linkage does not guarantee the correctness of the model of the entire system constructed in this way. A model built without taking into account the relationships of the system with the environment and its behavior in relation to this environment can often only serve as another confirmation of Gödel's theorem, or rather, its corollary, which states that in a complex isolated system there can be truths and conclusions that are correct in this system and incorrect outside it.

The science of modeling consists in dividing the modeling process (systems, models) into stages (subsystems, submodels), a detailed study of each stage, relationships, connections, relationships between them and then effectively describing them with the maximum possible degree of formalization and adequacy. If these rules are violated, we get not a model of the system, but a model of "own and incomplete knowledge".

Modeling (meaning "method", "model experiment") is considered as a special form of experiment, an experiment not over the original itself (this is called a simple or ordinary experiment), but over a copy (substitute) of the original. The isomorphism of systems (original and model) is important here - isomorphism of both the copy itself and the knowledge with which it was proposed.

Conclusion

Models and modeling are applied in the main areas:

- ✓ *training (both models, modeling, and the models themselves);*
- ✓ *cognition and development of the theory of the systems under study (using any models, modeling, modeling results);*
- ✓ *forecasting (output data, situations, system states);*
- ✓ *management (the system as a whole, individual subsystems of the system), the development of management decisions and strategies;*
- ✓ *automation (system or individual subsystems of the system).*

Control questions

See the manual on the organization of students' independent work.